# Dear T-Rex

### Addition is totally commutative, don't worry. I will show you.

| | | |
|---|---|---|
| 1) | $S(x) \neq 0$ | Axiom |
| 2) | $S(x) = S(y) \Rightarrow x = y$ | Axiom |
| 3) | Induction Schema | Axiom |
| 4) | $x + 0 = x \wedge x + S(y) = S(x + y)$ | Axiom |
| 5) | $0 + 0 = 0 = 0 + 0$ | from 4 |
| 6) | Assume $j + 0 = 0 + j$ | for some $j \in \mathbb{N}$ |
| 7) | $0 + S(j) = S(0 + j)$ | from 4 |
| 8) | $= S(j + 0)$ | from 6 |
| 9) | $= S(j)$ | from 4 |
| 10) | $= S(j) + 0$ | from 4 |
| 11) | $0 + S(j) = S(j) + 0$ | from 7 -10 |
| 12) | $j + 0 = 0 + j \Rightarrow S(j) + 0 = 0 + S(j)$ | from 6-11 |
| 13) | $\forall n(n + 0 = 0 + n \Rightarrow S(n) + 0 = 0 + S(n))$ | from 12 |
| 14) | $\forall n(n + 0 = 0 + n)$ | from 13, 5, 3 |
| 15) | $S(0) + 0 = 0 + S(0)$ | from 14 |
| 16) | Assume $S(0) + j = j + S(0)$ | for some $j \in \mathbb{N}$ |
| 17) | $S(0) + S(j) = S(0) + S(j + 0)$ | from 4 |
| 18) | $= S(0) + j + S(0)$ | from 4 |
| 19) | $= j + S(0) + S(0)$ | from 16 |
| 20) | $= S(j + 0) + S(0)$ | from 4 |
| 21) | $= S(j) + S(0)$ | from 4 |
| 22) | $S(0) + S(j) = S(j) + S(0)$ | from 17-21 |
| 23) | $S(0) + j = j + S(0) \Rightarrow S(0) + S(j) = S(j) + S(0)$ | from 16 -22 |
| 24) | $\forall n(S(0) + n = n + S(0) \Rightarrow S(0) + S(n) = S(n) + S(0))$ | from 23 |
| 25) | $\forall n(S(0) + n = n + S(0))$ | from 24, 15, 3 |
| 26) | Assume $\forall n(n + k = k + n)$ | for some $k \in \mathbb{N}$ |
| 27) | $j + S(k) = S(j + k)$, for some $j \in \mathbb{N}$ | from 4 |
| 28) | $= S(k + j)$ | from 26 |
| 29) | $= k + S(j)$ | from 4 |
| 30) | $= k + S(j + 0)$ | from 4 |
| 31) | $= k + j + S(0)$ | from 4 |
| 32) | $= k + S(0) + j$ | from 25 |
| 33) | $= S(k + 0) + j$ | from 4 |
| 34) | $= S(k) + j$ | from 4 |
| 35) | $j + S(k) = S(k) + j$ | from 27-34 |
| 36) | $\forall n(n + S(k) = S(k) + n)$ | from 35 |
| 37) | $\forall n(n + k = k + n) \Rightarrow \forall n(n + S(k) = S(k) + n)$ | from 26-36 |
| 38) | $\forall m(\forall n(n + m = m + n) \Rightarrow \forall n(n + S(m) = S(m) + n))$ | from 37 |
| 39) | $\forall m(\forall n(n + m = m + n))$ | from 38, 14, 3 |

**The End**

# Love, Alison