How Does Spider-Man Move so Fast?

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Abstract

Since time immemorial, mankind has wondered not only at Spider-Man's great agility, but also at his ability achieve large average velocities as he traverses the cavern city of Manhattan. How does he achieve these speeds merely by swinging by a thread? What is the nature of the filament from which he swings, and to what degree does it have an effect on his average horizontal velocity? We begin this paper with a calculation demonstrating that if Spider-Man's filament has a constant length, the average horizontal velocity is only comparable to that of a person on a bicycle (or alternatively, an automobile in a school zone). We then construct a model for a flexible filament which contracts immediately upon anchoring itself to a building. While the mechanism of this contraction and the specific values of the two parameters governing this model are, at this time, a matter of speculation; if we conjecture reasonable values for these two parameters, it is shown that Spider-Man's average horizontal velocity can attain that comparable to a subway.

1 Introduction

How is it that Spider-Man travels so rapidly across the Manhattan skyline? He begins at a great height, affixed to the face of a building; and he somehow projects a filamentary "web" above and beyond him in the direction he wishes to travel which anchors itself to another tall building. Upon taking to the air, he swings down and forward —his gravitational potential energy from his drop in altitude is converted to forward moving kinetic energy. In midair, he projects and transfers himself to another filament which he has anchored further down the street. How amazing his long-limbed, inverted strides seem to those of us crawling along the pavement on two short legs.

At the bottom of his swing he can surely achieve some outstanding velocities; however, these photo-blurring speeds are not maintained for the duration of his swing. Should Spider-Man rely only upon his initial gravitational potential energy for locomotion, traversing the city upon a series of pendulums, then his resulting average velocity would be deceptively small. The period of a pendulum, after all, increases with its length; and Spider-Man's filaments can be up to one hundred meters long!

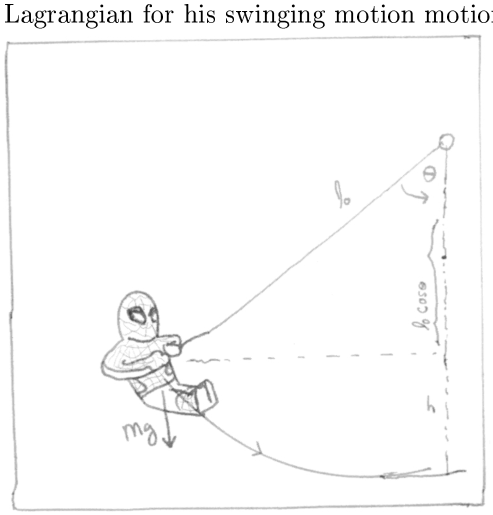
We construct an alternate model for Spider-Man's locomotion which will allow him to achieve much faster average horizontal velocities. In this model, we treat a filament which has initial length l_o (the distance from his hand to the anchor on the building) as a spring which has been stretched a distance $l_o(1-\lambda)$ (where $\lambda < 1$). The restoring force of the spring will pull him forward, and allow him to achieve an average velocity comparable to a subway.

2 Swinging Along a Filament of Constant Length

The simplest model of Spider-Man's locomotion involves a web whose length remains effectively constant for the duration of his swing. In this section, we will outline the drawback of this model: the average horizontal velocity is far too low.

2.1 Pendulum Motion

If we suppose that Spider-Man's filament maintains a constant initial length l_o, the Lagrangian for his swinging motion motion would be



$$\mathcal{L} = \frac{1}{2} m_{SM} l_o^2 \dot{\theta}^2 - m_{SM} g h = \frac{1}{2} m_{SM} l_o^2 \dot{\theta}^2 - m g l_o (1 - \cos\theta)$$

which leads to the equation of motion:

$$\ddot{\theta} = -g \frac{1}{l_o} \sin\theta \,.$$

If we assume that θ is relatively small, we can approximate $\sin\theta \approx \theta$, and the equation of motion can be approximated

$$\ddot{\theta} = -\frac{g}{l_o} \theta$$

which is the equation of motion of a harmonic oscillator. Therefore, the period of one swing will be:

$$T = 2\pi\sqrt{\frac{l_o}{g}} .$$

2.2 Maximum Average Velocity

So long as the small angle approximation holds, the period of the swing will be independent of how wide the swing is. If we suppose that the $\theta = \frac{\pi}{6}$, the horizontal width subtended by a swing will be:

$$W = 2l_o \sin(\frac{\pi}{6}) = l_o .$$

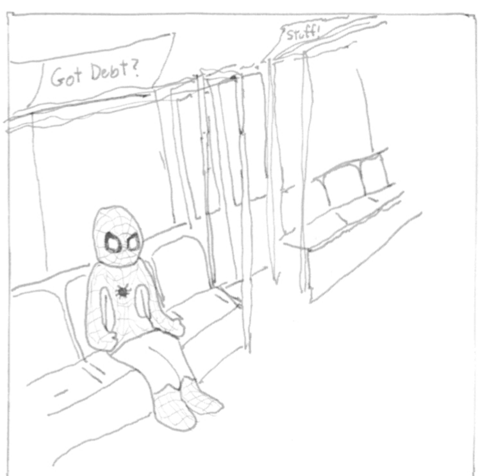
In this case, the average horizontal velocity across the swing will be

$$V_{AV} = \frac{W}{\frac{1}{2}T} = \frac{\sqrt{l_o g}}{\pi} .$$

We see that since the average speed *increases* with the square root of the length of the web, Spider-Man achieves his fastest possible speed by taking the longest possible swings he can. If we suppose that the maximum length of webbing emitted by Spider-Man has a length of $100m$ (about the length of a football field), Spider-Man's average horizontal velocity will be

$$V_{AV} = 9.97\frac{m}{s} = 35.9\frac{km}{hr}$$

which is comparable to the speed of someone on a bicycle; and is much slower than taking the subway. Indeed, it's not clear why, if he were limited to such a slow speed, he would not use the efficient Manhattan mass transit system or simply hail a cab.



3 The Dynamic Web Model

Since Spider-Man clearly travels faster than his gravitational potential energy alone will allow; it is clear that he must be using the elastic properties of his webbing to achieve a higher average velocity. We conjecture that the elasticity of Spider-Man's web can be approximated by Hooke's law; and that, even at the length at which it emerged from his wrist, it would be in a stretched state. We speculate that to have started out as a stretched spring, the filament must (upon emerging from Spider-Man's wrist) undergo some rapid chemical reaction, where the work performed by the chemical reaction is converted into the tension in the filament.

3.1 Self Stretching Webbing

3.1.1 Parametrizing the Stretching Effect

Our model conjectures that if we were to ask Spider-Man to excrete a filament one meter in length and place it upon a laboratory table untethered: we would immediately see it shrink to a few centimeters in length. We suppose that final length l_f to which the filament shrinks will therefore be proportional to the the initial length l_o:

$$l_f = \lambda l_o.$$

If we were to repeat this procedure, asking Spider-Man to provide us with another segment of filament of length l_o — and this time we were to secure the ends of the filament so that it retained its initial length— we conjecture that the tension in the filament would quickly increase, and that the tension force in the filament would satisfy Hooke's law:

$$F_T = k(l - \lambda l_o)$$

where k is the spring constant and l is the current length of the filament.

We finally conjecture that the cause of this length contraction is a chemical reaction occurring within and along the web filament; possibly triggered by it's exposure to air. In this case, prior to the chemical reaction, the filament of length l_o would have a chemical potential energy density σ and total chemical potential energy $E = \sigma l_o$. If, as it shrank, the length of the filament l_o were held constant; the chemical energy would then be converted into the potential energy required to stretch the filament from length λl_o to length l_o:

$$l_o \sigma = \frac{1}{2} k l_o^2 (1-\lambda)^2 \; .$$

Thus, the spring constant can be rewritten in terms of the chemical energy density σ, the initial filament length l_o, and the shrinking ratio λ

$$k = \frac{2\sigma}{l_o(1-\lambda)^2} \; .$$

Note that the stiffness of the filament will depend inverse proportionally on its initial length: the longer the filament is initially, the stretchier it will be. Finally, note that a web of initial length l_o and current length l will exert a tension force

$$\vec{F} = \frac{2\sigma}{l_0(1-\lambda)^2}(l - l_o\lambda)\vec{r}$$

where $\vec{r}$ is a normalized vector pointing from Spider-Man to the anchor point of the web. This will be referred to as the *Dynamic Webbing* model.

3.1.2 Selecting Values for the Parameters σ and λ

Fixing the values for λ and σ aught to be a matter of measurement: moments of study in a lab, or a few minutes of high quality video would be sufficient to provide them.

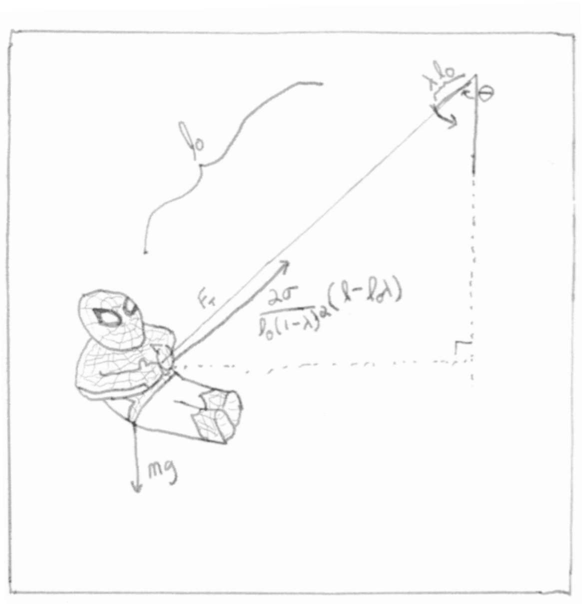
Without these opportunities, we are forced to speculate; and for this paper, we will suppose that $\lambda = 0.01$. This effectively decouples the force of gravity from Spider-Man's horizontal movement, and it also ensures that any web which is not weighed down will shrink to a small fraction of its initial length.

It seems reasonable to us that Spider-Man should be able to hang vertically from a fresh web without having it either stretch or contract dramatically. Thus, the force of tension from a filament should be nearly equal to Spider-Man's weight $F_g = F_T$:

$$mg = \frac{2\sigma}{(1-\lambda)^2 l_o}(l_o - l_o\lambda) = \frac{2\sigma}{(1-\lambda)}\ . \tag{1}$$

3.2 Dynamics of Swinging on a Dynamic Web Filament

It is simpler in this case to deal with the dynamics in terms of Newtonian forces and Cartesian coordinates, rather than generating the equations of motion through the Lagrangian formalism in terms of degrees of freedom.



$$F_x = \ddot{x}m = -\frac{2\sigma}{l_o(1-\lambda)^2}(l - l_o\lambda)\sin\theta = \frac{2\sigma}{l_o(1-\lambda)^2}x(1 - \frac{\lambda l_o}{l})$$

$$F_y = \ddot{y}m = \frac{2\sigma}{l_o(1-\lambda)^2}(l - l_o\lambda)\cos\theta - gm = \frac{2\sigma}{l_o(1-\lambda)^2}y(1 - \frac{l_o\lambda}{l}) - gm \ .$$

Since the unstretched length λl_o will be very small compared to the current length l, the effective $x(t)$ equation of motion can be written

$$\ddot{x} \approx \frac{2\sigma}{m(1-\lambda)^2 l_o}x \ .$$

Thus the period of Spider-Man's horizontal motion will be

$$T = 2\pi\sqrt{\frac{l_o(1-\lambda)^2 m}{2\sigma}}$$

and the average horizontal velocity:

$$V_{av} = \frac{\sin(\theta_o)l_o}{\frac{1}{4}T} = \frac{2\sqrt{2\sigma l_o}}{\pi(1-\lambda)\sqrt{m}}\sin(\theta_o) \ .$$

From here on, we'll approximate $\lambda \approx 0$, and according to equation (1) we'll suppose that $mg \approx 2\sigma$. The average horizontal speed will be

$$V_{av} \approx \frac{2\sqrt{gl_o}}{\pi}\sin(\theta_o) \ .$$

The maximum angle in this case will be a swing from $\theta_o = \frac{\pi}{2}$, and if the maximum initial filament length is $l_o = 100m$: Spider-Man's maximum average velocity will be:

$$V_{Max} \approx 20\frac{m}{s} = 70\frac{km}{hr}$$

which is comparable to the speed of a subway.

We should note that, should Spider-Man emit a web from both his wrists, doubling the tension of the web pulling him forward, he would be increasing his average velocity by a factor of $\sqrt{2}$, providing an average velocity of nearly $100\frac{km}{hr}$. The benefit of the dynamic webbing system is clearly that by increasing the number of filaments, he is able increase the tension available to him and thus, his speed.

4 Conclusion

It is clear that the model for Spider-Man's web wherein the length of a filament remains nearly constant is inadequate: even if Spider-Man were able to emit and control a filament 100m in length, it would not be sufficient to travel more quickly through the city than a bicycle courier. We have constructed the dynamic webbing model wherein Spider-Man is provided, at the beginning of each swing, with a filament which acts effectively as a stretched spring. The initial energy provided by the stretched filaments pulls him forward to achieve greater average speeds. Supposing that initial the tension in a filament is sufficient to hold Spider-Man aloft, we demonstrate that he should be able to attain speeds comparable to motorized means of transportations. Furthermore, he has the ability to achieve even faster speeds by using more than one filament at a time.

I would like to thank Johan Brannlund, Miles Steininger, and Ryan North for their help.

Appendix: Gwen Stacy's death

The matter of Gwen Stacy's death remains a controversy: did she die of whiplash due to Spider-Man's own negligence, or did she succumb to some other accident or violence prior or during the fall? With our dynamic webbing model, we have the capacity to examine the question of how much acceleration she felt as she fell from the 184m height of the George Washington bridge.

Suppose that Gwen Stacy fell 91 meters (300 feet) before Spider-Man was able to secure a web to her leg. The web would then stretch, slowing her fall, until she stopped. To determine the distance from the top of the pillar to the height at which Miss Stacy stopped falling, we note that the energy required to the stretch the filament from $l_o = 91m$ to the final depth of l_f would be equal to the total gravitational potential energy of Miss Stacy's fall to that depth:

$$m_{GS} l_f g = \frac{2\sigma}{l_o(1-\lambda)^2} \int_{l_o}^{l_f} (x - l_o\lambda) dx$$

$$m_{GS} l_f g = \frac{2\sigma}{l_o(1-\lambda)^2}(\frac{1}{2}l_f^2 - \frac{1}{2}l_o^2 - l_o\lambda l_f + l_o^2\lambda) \ .$$

Our previous assumptions will let us approximate the values of σ and λ: $\sigma \approx mg/2$, $\lambda \approx 0$, and we can solve for the distance to the turning point l_f:

$$l_f = 91(\frac{m_{GS}}{m_{SM}} + \sqrt{1 + (\frac{m_{GS}}{m_{SM}})^2}) \ .$$

If Gwen Stacy's is mass is $m_{GS} = 50$kg and Spider-Man's mass is $m_{SM} = 73Kg$, the distance fallen will be:

$$l_f = 172m$$

which is still 12 meters short of hitting the water.

The question is now: what is the maximum force applied to Gwen Stacy as the dynamic webbing slows her fall? The force from the web is given by:

$$F_T = \frac{2\sigma}{l_o(1-\lambda)^2}(l_f - l_o\lambda) = 1353N$$

and since Miss Stacy only weighs $m_{GS}g = 490N$; the tension force is less than 3X her body weight. Clearly, being caught by Spider-Man's web, even by the foot, is no more dangerous than going bungee jumping. We conclude that it was probably not the tension of the web alone which caused Gwen Stacy's unfortunate passing.